Lesson 8 - Integration By Parts - Part II

I. Recall
II Multiple rounds of By Parts

The Lesson
I. Recall
   By Parts Formula

$$\int u\,dv = uv - \int v\,du$$

   Choosing u

$$
\begin{aligned}
&L \text{ ogs} \\
&A \text{ lgebraic } (\text{Polynomials/ Roots}) \\
&T \text{ rig} \\
&E \text{ xponential}
\end{aligned}
$$

New Today?. Sometimes we need multiple
            rounds of By Parts

II. Multiple Rounds of By Parts

EX $\int x^2 \cos(x)\, dx = x^2 \sin(x) - \int 2x \sin(x)\, dx$

Rd 1

$u = x^2 \qquad dv = \cos(x)\, dx$

$du = 2x\, dx \qquad \int dv = \int \cos(x)\, dx$

$\qquad\qquad\qquad v = \sin(x)$

$= x^2 \sin(x)$
$\quad - \left[2x(-\cos(x)) - \int -2\cos(x)\, dx \right]$

$= x^2 \sin(x)$
$\quad - \left[-2x\cos(x) + \int 2\cos(x)\, dx \right]$

Rd 2

$u = 2x \qquad dv = \sin(x)\, dx$

$du = 2\, dx \qquad \int dv = \int \sin(x)\, dx$

$\qquad\qquad\qquad v = -\cos(x)$

$= x^2 \sin(x)$
$\quad - \left[-2x\cos(x) + 2\sin(x) \right] \qquad +C$

$= x^2 \sin(x) + 2x\cos(x) - 2\sin(x)$
$\qquad\qquad\qquad +C$

EX $\int_{3} 4x^2 e^{5x}\, dx$

$\int 4x^2 e^{5x}\, dx = 4x^2 \cdot \tfrac{1}{5} e^{5x} - \int \tfrac{1}{5} e^{5x} \cdot 8x\, dx$

Rd 1:

$u = 4x^2 \qquad dv = e^{5x}\, dx$

$du = 8x \qquad \int dv = \int e^{5x}\, dx = \int e^u \cdot \tfrac{1}{5} du$

$\qquad\qquad\qquad b = 5x$
$\qquad\qquad\qquad db = 5\, dx \qquad = \tfrac{1}{5} e^u$
$\qquad\qquad\qquad \tfrac{1}{5} db = dx \qquad = \tfrac{1}{5} e^{5x}$

$\qquad\qquad v = \tfrac{1}{5} e^{5x}$

$= \tfrac{4}{5} x^2 e^{5x} - \int \tfrac{8}{5} x e^{5x}\, dx$

$= \tfrac{4}{5} x^2 e^{5x}$
$\quad - \left[\tfrac{8}{25} x e^{5x} - \int \tfrac{8}{25} e^{5x}\, dx \right]$

$= \tfrac{4}{5} x^2 e^{5x}$
$\quad - \left[\tfrac{8}{25} x e^{5x} - \tfrac{8}{25} \cdot \tfrac{1}{5} e^{5x} \right]$

$\qquad\qquad +C$

Rd 2:

$u = \tfrac{8}{5} x \qquad dv = e^{5x}\, dx$

$du = \tfrac{8}{5}\, dx \qquad v = \tfrac{1}{5} e^{5x}$

$$\int 4x^2 e^{5x} dx = \frac{4}{5}x^2 e^{5x} - \frac{8}{25}xe^{5x} + \frac{8}{125}e^{5x} + C$$

$$\int_0^3 4x^2 e^{5x} dx = \frac{4}{5}(3)^2 e^{5\cdot3} - \frac{8}{25}(3)e^{5\cdot3} + \frac{8}{125}e^{5\cdot3}$$

$$- \left[\frac{4}{5}(0)^2 e^{5\cdot0} - \frac{8}{25}(0)e^{5\cdot0} + \frac{8}{125}\underbrace{(e^{5\cdot0})}_{=1} \right]$$

$$= \frac{36}{5}e^{15} - \frac{24}{25}e^{15} + \frac{8}{125}e^{15} - \frac{8}{125}$$